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The graphical interpretation of fracture load data for doubly-convex cylindrical discs

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A set of graphs is developed for the determination of the material tensile strength of a brittle, doubly-convex cylindrical disc from the disc dimensions and the fracture load in in-plane diametral compression. Graphs are also presented relating the normalized geometric variables of the disc and giving the normalized volume of the disc in terms of the geometric parameters.

1. Introduction

The symmetrical doubly-convex cylindrical disc (Fig. 1) is the generic form of a wide range of brittle compacts, typified for example by a large number of pharmaceutical tablets. In designing such compacts and appraising their quality, a means of determining the material strength (or "hardness") and an understanding of the relationship between geometric parameters and material strength are important requirements. The "Brazilian disc" test or "indirect" tensile test [1], in which the disc is loaded to fracture by means of two equal, opposed, in-plane compressive forces, has an invaluable role in this context and is widely used. For the plane-faced disc, classical theory of elasticity [2] provides a simple relationship between the fracture load and the material tensile strength. For the convex-faced disc, as a result of fracture studies of brittle specimens, a relationship between fracture load (P_s), geometric variables and material tensile strength (σ_f) has been developed [3] in the form

$$\sigma_f = \frac{10P_s}{\pi D^2 F} \qquad (1)$$

where

$$F = 2.84\frac{t}{D} - 0.126\frac{t}{W} + 3.15\frac{W}{D} + 0.01 \qquad (2)$$

and t is the overall thickness of the convex-faced disc (see Fig. 1), D is the diameter and W is the length of the cylindrical portion.

The range of variables covered in establishing Equation 1 was

$$0.06 \leqslant \frac{W}{D} \leqslant 0.3$$

$$0 \leqslant \frac{D}{R} \leqslant 1.43$$

where R is the radius of curvature of the disc faces.

Equation 1 is definitive in that the term F relates uniquely and specifically to the disc dimensions, but the variables in the equation are not the most relevant and in the form given the equation does not lend itself to speedy interpretation. In this paper, Equation 1 is remodelled and presented in graphical form in terms of the variables t/D and D/R, so that fracture load results for a disc of a given shape can be readily converted into material tensile strength values. Other graphical aids are also given in the paper.

2. Graphical approach

In considering alternative forms of Equation 1, it was accepted that the most useful non-dimensional parameters are R/D and t/D. The former follows directly from the punch and disc dimensions, which will be known, and the thickness t can be readily measured, using a micrometer for example. The following relationship between the dimensional variables shown in Fig. 1 is readily derived:

$$W = t - [2R - (4R^2 - D^2)^{1/2}] \qquad (3)$$

i.e.

$$\frac{W}{D} = \frac{t}{D} - f\left(\frac{R}{D}\right) \qquad (4)$$

where

$$f\left(\frac{R}{D}\right) = \frac{2R}{D} - \left(\frac{4R^2}{D^2} - 1\right)^{1/2} \qquad (5)$$

Using Equation 3, W can be eliminated from F (Equation 2) to give

$$F = 5.99\frac{t}{D} - 0.126\left[1 - \frac{D}{t}f\left(\frac{R}{D}\right)\right]^{-1} - 3.15f\left(\frac{R}{D}\right) + 0.01 \qquad (6)$$

The term F has been calculated from Equation 6 for a series of t/D values (in intervals of 0.05) over the range of D/R values $0 \leqslant D/R \leqslant 1.6$. The results are shown in the first quadrant of Fig. 2 (top right) in the form of a graph of F against D/R for each t/D value. The lower and upper t/D limits of the experimental test range for a given D/R value are readily calculated from Equation 4 using the limiting W/D values of 0.06 and 0.3 in turn. The corresponding F values may then be obtained from Equation 6. The derived lower and upper limits of the experimental test range are shown as

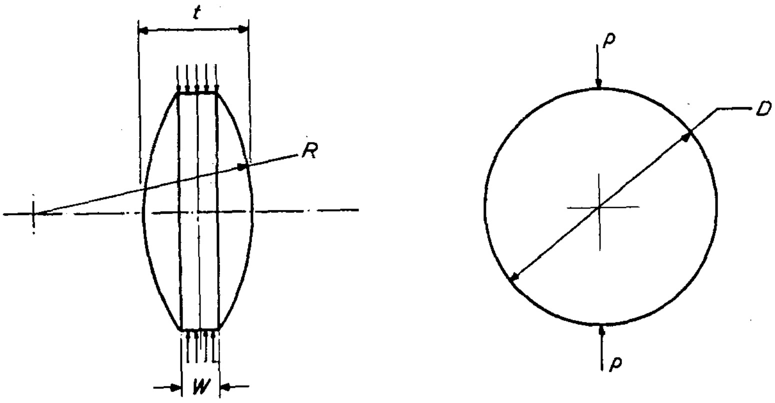

Figure 1 Doubly-convex cylindrical disc subjected to diametrically opposed compressive forces p.

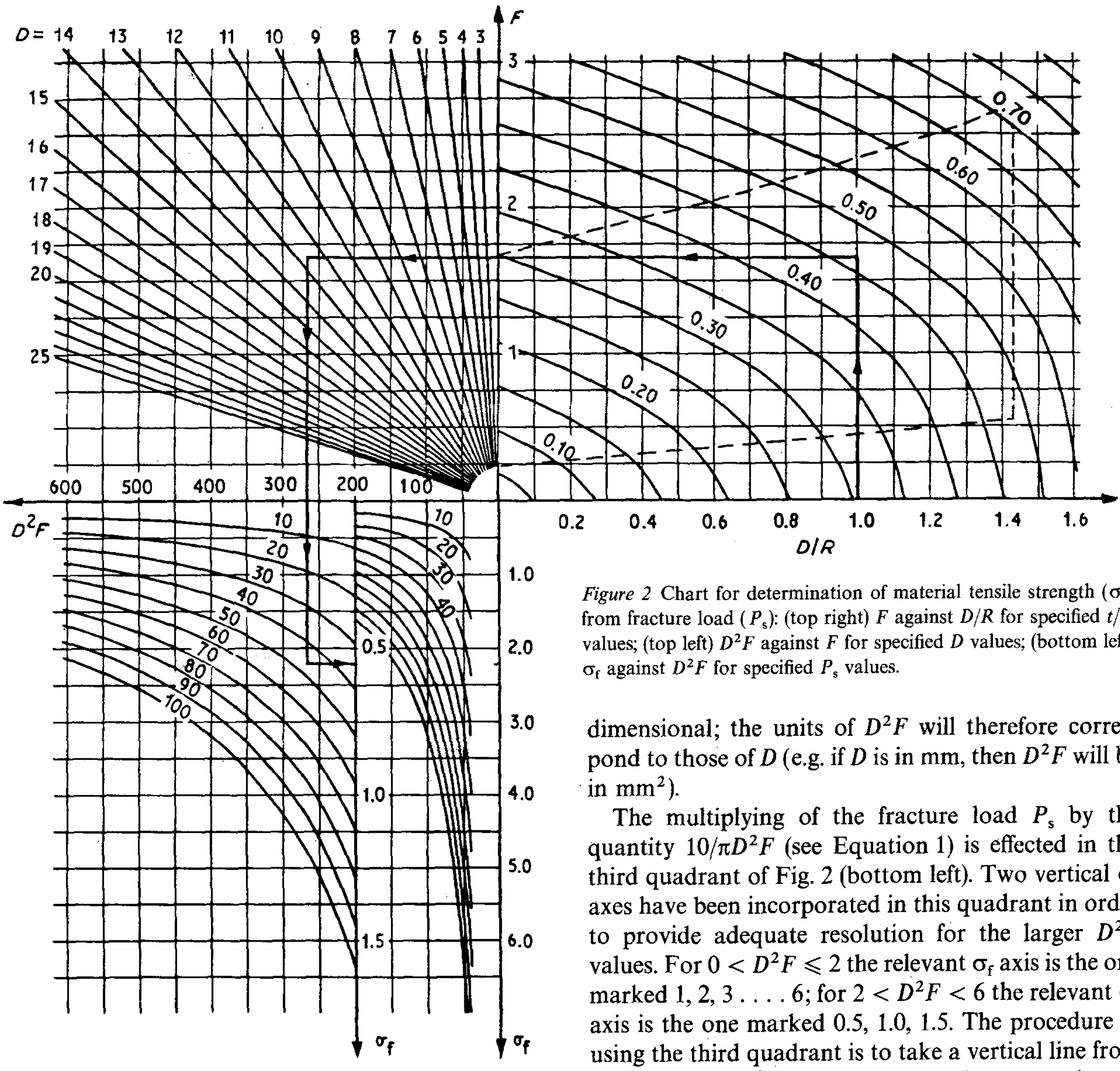

Figure 2 Chart for determination of material tensile strength (σ_f) from fracture load (P_s): (top right) F against D/R for specified t/D values; (top left) D^2F against F for specified D values; (bottom left) σ_f against D^2F for specified P_s values.

broken lines in Fig. 2, and, strictly speaking, the validity of the empirical Equation 2 is restricted to the shapes within the enclosed area. Cautious extrapolation outside this, however, should not introduce serious errors.

The second quadrant of Fig. 2 (top left) provides a graphical means of multiplying the quantity F by D^2. If a horizontal line is drawn from a specified point on the vertical axis (labelled F) on to the appropriate D line, then a vertical line from the point of intersection will intersect the horizontal axis (labelled D^2F) at a point corresponding to the product D^2F. F is non-dimensional; the units of D^2F will therefore correspond to those of D (e.g. if D is in mm, then D^2F will be in mm^2).

The multiplying of the fracture load P_s by the quantity $10/\pi D^2F$ (see Equation 1) is effected in the third quadrant of Fig. 2 (bottom left). Two vertical σ_f axes have been incorporated in this quadrant in order to provide adequate resolution for the larger D^2F values. For $0 < D^2F \leqslant 2$ the relevant σ_f axis is the one marked 1, 2, 3 . . . . 6; for $2 < D^2F < 6$ the relevant σ_f axis is the one marked 0.5, 1.0, 1.5. The procedure in using the third quadrant is to take a vertical line from a point on the D^2F axis down on to the appropriate P_s curve and then a horizontal line to intersect the appropriate σ_f axis; the point of intersection gives the material tensile strength or "hardness", σ_f. The units of σ_f will correspond with those of D and P_s. If these are millimetres and newtons respectively, then σ_f will be in N mm^{-2} (i.e. MPa); if P_s is obtained in kilograms (which is not strictly a unit of force) then, with D in millimetres, σ_f will be in kg mm^{-2}. Curves corresponding to only one decade of P_s values (from 10 to 100) are shown but a multiplying, or dividing, factor of 10 (or any multiple of 10) can be readily introduced into the P_s curves and carried forward to the σ_f axes.

To illustrate the use of the Fig. 2 curves, an example is shown for which $D = 12.5$ mm, $R = 12.5$ mm (i.e. $D/R = 1.0$), $t = 5.85$ mm ($t/D = 0.468$) and $P_s = 45.3$ kg. With these data, the F value obtained in the first quadrant is 1.67. (Linear interpolation between adjacent t/D curves does not generally introduce significant error.) The D^2F value obtained in the second quadrant is 260 mm^2 (again with linear interpolation between adjacent D lines). Linear interpolation along a vertical line through the P_s curves in the third quadrant involves no approximation, and a σ_f value of 0.55 kg mm^{-2} (i.e. 54.1 MN m^{-2}) is obtained. The same value is obtained by direct calculation from Equations 1 and 2.

2.1. Other graphs

Two further graphs are presented to facilitate data processing in this context.

Fig. 3 is a graphical version of Equation 4, from which W/D may be readily derived for known values of t/D and D/R, or any one normalized variable obtained in terms of the other two. Equation 4 is a geometrical relationship and is valid without limit for all positive values of the variables shown in Fig. 1.

The volume, V, of a doubly-convex disc is an essential requirement in the determination of the material density. It is readily shown that

$$V = \frac{\pi D^3}{4}\left[\frac{t}{D} - g\left(\frac{R}{D}\right)\right] \qquad (7)$$

where

$$g\left(\frac{R}{D}\right) = f\left(\frac{R}{D}\right)\left\{1 - f\left(\frac{R}{D}\right)\left[\frac{2R}{D} - \frac{1}{3}f\left(\frac{R}{D}\right)\right]\right\} \qquad (8)$$

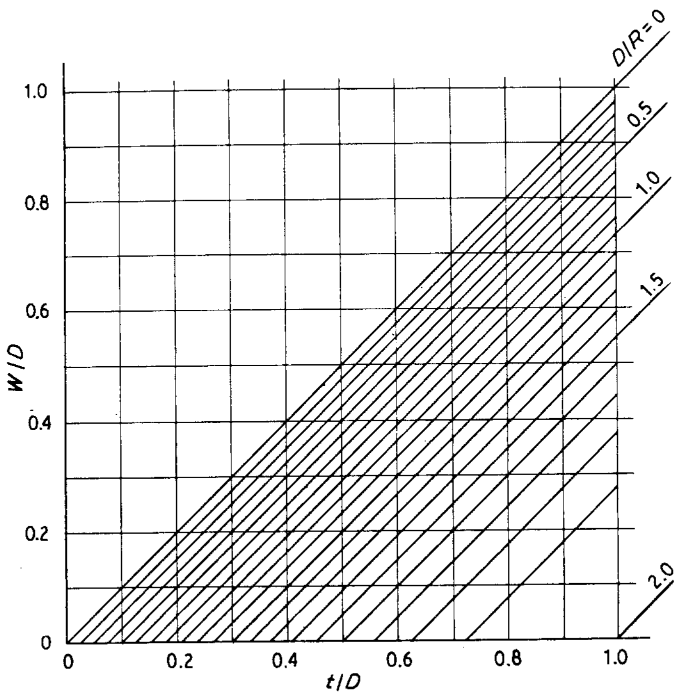

Figure 3 Ratio of cylinder length to diameter (W/D) against ratio of overall thickness to diameter (t/D).

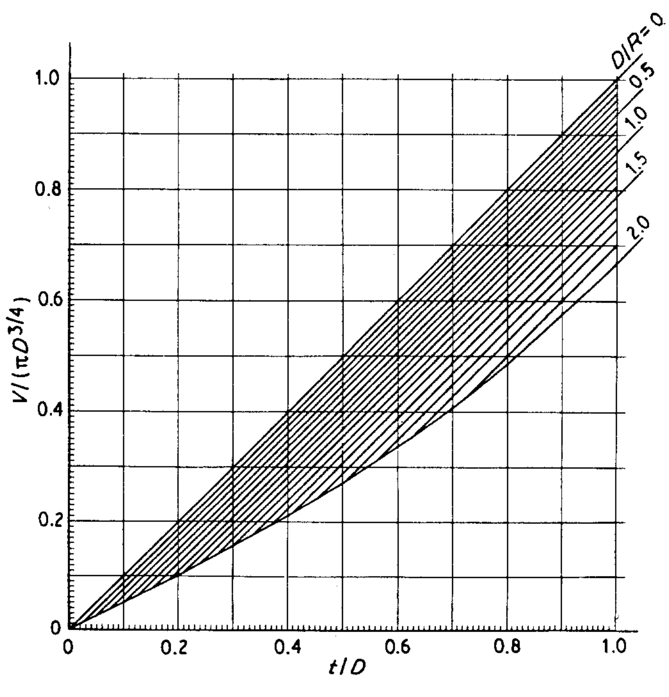

Figure 4 Normalized volume ($V/(\pi D^{3/4})$) against ratio of overall thickness to diameter (t/D).

or alternatively

$$\frac{V}{(\pi D^3/4)} = \frac{t}{D} - g\left(\frac{R}{D}\right) \qquad (9)$$

In Equation 9 the volume V is normalized with respect to $\pi D^3/4$, the volume of a plane-faced disc (i.e. $D/R = 0$) with $t = D$. $V/(\pi D^3/4)$ is plotted in Fig. 4 against t/D for a series of D/R values and the normalized volume can be readily read from the graph when t/D and D/R are known. For a specified D the volume is then calculated directly. The lower bounding curve in Fig. 4 results from the condition that W, the length of the cylindrical portion of the disc, must be equal to, or greater than, zero. With this proviso, and positive values of the other variables, Equation 9, like Equation 4, is valid without limit.

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